Investment Shocks and the Optimal Monetary Policy

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Abstract

Despite the importance that such a shock entails as a driver of the business cycles, optimal monetary policy with investment-specific shock is rarely touched by the literature. Using a calibrated DSGE model with nominal rigidities, we explore how central banks should respond to two different types of investment shocks: shocks to production of investment goods (IST) and shocks to the marginal efficiency of investment (MEI). Our main findings are: First, the type of investment shocks determines the shape of optimal monetary policy. Monetary policy conducted indiscriminate to the type of shocks causes severe welfare losses. Second, optimal monetary policy takes the form of targeting a weighted average of consumption and investment goods inflations. Third, the form of the optimal monetary policy depends significantly on the mobility of production factors. When factors are not allowed to be reallocated across sectors, both shocks generate tradeoffs, while only IST shocks cause the tradeoff under perfect factor mobility condition. Forth, the optimal weights on each sectoral inflation depends on model parameters, especially those governing labor elasticity and depreciation rate of investment goods. Our results suggest that the central bank should not only focus on the underlying nature of the investment shocks, but also has to estimate the key parameters precisely to conduct a appropriate monetary policy.

Key Words: Optimal monetary policy, IST shock, factor mobility, output gap targeting
JEL Classification:

1 Introduction

Since the seminal paper by Smets and Wouters (2007), investment shocks have received considerable attention as a driver of the business cycles. Incorporating investment shocks into a DSGE model takes two major ways: one approach assumes and investment-specific technology (IST) shock, and the other approach assumes a shock to marginal efficiency of investment (MEI) (Justiniano et al., 2010). The former could be represented by a shock to the investment goods production function, while the latter is formalized as a shock to the evolution of capital stock. Early study of Greenwood et al. (2000) showed that the two approaches are equivalent in a special environment, and a large body of following literature adopted Greenwood et al. (2000) in modelling investment shocks.[1]

However, when studying monetary policy toward investment shocks is the main concern of the research, this conventional approach can be problematic because it does not distinguish the consumption inflation and the investment inflation. Because IST shocks are sector specific supply shock, it would create a disproportionary effect between the two sector. On the other hand, as MEI shocks affect the accumulation of aggregate capital, the impact of such shocks would be symmetric to multiple sectors. As pointed out in Erceg and Levin (2006), welfare loss function in multi-sector economy is decomposed into sectoral variables, with inflations in each sector treated separately. In this situation, misspecification of investment shocks would result in a mishandled monetary policy towards such shocks, resulting in unnecessary welfare losses.

Our study is intrigued by the idea that those two kinds of investment shocks can possibly require different form of monetary policy. Specifically, we focus on the different propagation mechanism of those two shocks under nominal rigidities. Nominal rigidities such as price stickiness make relative price of investment distorted, and produce different progataions of IST and MEI shocks. In our setting, those two shocks require starkly different monetary policy, and misperception on the nature of shocks can generate welfare losses to a considerable extent. When we classify investment shocks into those two categories, whether the production factors can move across sectors can potentially have an important implication. When factors are freely mobile in multiple sectors, demand shocks such as MEI shock could be dealt easily by the *divine coincidence* result. However, when factors are not allowed to move across sectors, even shocks to the aggregate economy could entail assymmetric effects because input factors are tied up in its owns sector. So far, a large body of literature studying optimal monetary policy assume either perfect mobility or immobility of input factors. Erceg and Levin (2006) investigated the form of optimal monetary policy when there is consumer durable sector, assuming labor forces tied up to a specific sector. Barsky et al. (2016) also studied the optimal monetary policy with durable goods, also assuming factors are not allowed to move across sectors. Basu et al. (2016) analyze a multi-sector DSGE model which shares many features of our model, again assuming labor forces are free to move across sectors. Bils et al. (2013) also construct a multi-sector DSGE model to study the so-called "Keynesian Labor Demand", this time limiting the degree to which factors can move between sectors.

[1] For the empirical consistency of this modelling approach, please refer to Guerrieri et al. (2014).

Liu et al. (2012) also considers two sector DSGE model with nominal rigidities to analyze the impact of investment shocks, but they also assumes that factors are mobile across sectors.

Unlike those previous studies, we analyze the optimal monetary policy towards investment shocks in a broader perspective, nesting the importance of factor mobility. We construct two versions of two-sector calibrated DSGE model. Our benchmark model allows factors freely move across sectors. On the other hand, we also construct a model with imperfect factor mobility, leaving other parts of the model unchanged. Regarding the two sector DSGE model with imperfect factor mobility, Carvalho and Nechio (2016) proposed a similar model to ours, a multisector model in which factors are free to move only within sector and unable to move across sectors. Also, our approach of modelling factor immobility, especially labor immobility can be seen as a special case of the method suggested by Katayama and Kim (forthcoming). However, those studies did not focus in the monetary policy design. Using the models, we investigate the path of macroeconomic variables under the optimal monetary policy and compare the welfare costs of various monetary policy rules. We also inspect the role of the labor supply elasticities on the performance of monetary policy rules, which is another important contribution of our study.

We find that the composite inflation targeting rule, which targets a weighted average of consumption sector and investment sector inflations attains highest level of welfare. The optimal weights attached to each sectoral inflation varies according to the degree of labor supply elasticity and the degree of factor mobility. Our results demonstrate that none of the conventionally conducted monetary policy, such as CPI targeting or output gap targeting, is universally successful. Sometimes, one monetary policy which is succesful towards IST shocks generate tremendous welfare losses when faced with MEI shocks. Moreover, we report that factor mobility and the degree of labor supply elasticity are crucial elements of monetary policy design. When factor immobility is introduced to the economy, monetary policies which were succeful for the IST shocks could be welfare deteriorating for MEI shocks. Especially, we find that CPI targeting produces tremendous welfare losses towards MEI shocks regardless the value of labor supply elasticity, when factors are tied up to their own sectors. This discrepancy is observed to be resolved as the labor supply elasticity rises. Our results suggest that the central bank should not only focus on the underlying nature of the investment shocks, but also has to estimate the key parameters precisely to conduct a appropriate monetary policy.

The rest of our paper is organized as follows. In the following section, we introduce two versions of calibrated DSGE models. The first model, which is our benchmark model, assumes perfect factor mobility. The latter model relax this assumption and limit the mobility of factors across sectors. Using those models, in section 3 we analyze the effects of investment shocks and investigate the diffence between IST and MEI shocks. In section 4 we evaluate various monetary policy rules towards investment shocks, and inspect the possible consequences of misuse of monetary policy. Our conclusion is summarized in section 5.

2 Model

We develop a model consists of two sectors that produce consumption goods and investment goods, calibrated to the U.S. economy. Production function in each sector is subject to both aggregate TFP shocks and sector-specific productivity fluctuations. Product markets are characterized by monopolistic competition and price rigidities, while labor market is perfectly competitive.

2.1 Model with perfect factor mobility

We first introduce our benchmark model with perfect input factor mobility. In this setting, sectoral labor and capital can move across sectors without any cost.

2.1.1 Households

We assume that there is a continuum of households endowed with differentiated labor, which is denoted with s. A household with labor type s maximized its lifetime utility

$$U_0 = E_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t, N_t)\right]$$
$$U(C_t, N_t) = \log C_t - b\frac{\eta}{1+\eta}N_t^{\frac{\eta+1}{\eta}}$$

where $\beta \in (0,1)$ is the subjective discount factor and η is a Frisch elasticity of aggregate labor supply. In each period of time, the household purchases consumption goods C_t at its price $P_{c,t}$, and investment good I_t at its price $P_{i,t}$. Therefore, the household has the following budget constraint:

$$P_{c,t}C_t(s) + P_{i,t}I_t(s) + B_t(s) + T_t \leq R_{t-1}B_{t-1}(s) + W_tN_t(s) + R_t^kK_t(s)$$

where $B_t(s)$ is nominal bond purchase by type s household, and T_t is a nominal lump-sum taxation. R_t is interest rate, which is policy instrument of central bank, and R_t^k and W_t are rental rate of capital and nominal wage, respectively.

Aggregate capital is accumulated according to the following equation.

$$K_{t+1} = \varphi_t I_t\left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] + (1-\delta)K_t \tag{1}$$

where κ captures the convex investment adjustment cost proposed by Christiano et al. (2005). We interpret the investment shock φ_t as a MEI shock, a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital and thus into tomorrow's capital input. It has the following stochastic process

$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \epsilon_{\varphi,t} \tag{2}$$

The first order conditions associated with the household's optimal choice of C_t, I_t, B_t and N_t are:

$$\frac{1}{C_t} = \lambda_t \tag{3}$$

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{c,t+1}} \right) \tag{4}$$

$$\begin{aligned} \lambda_t q_t = \mu_t \varphi_t & \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ & + \kappa \beta E_t \left[\mu_{t+1} \varphi_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \end{aligned} \tag{5}$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} \frac{R^k_{t+1}}{P_{c,t+1}} + (1-\delta) \mu_{t+1} \right] \tag{6}$$

$$b C_t N_t^{\frac{1}{\eta}} = w_t \lambda_t \tag{7}$$

where λ_t is the Lagrange multiplier attached to the household's budget constraint.

2.1.2 Firms

Final goods firms

A final good in each sector is produced by a representative firm in a perfectly competitive environment. The final good producing firm in each sector aggregate the intermediate goods with the constant elasticity of substitution (CES) technology.

$$C_t = \left[\int_0^1 C_t(s)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} ds \right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}} \tag{8}$$

$$I_t = \left[\int_0^1 I_t(s)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} ds \right]^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \tag{9}$$

where $\varepsilon_j > 1$ denotes the intratemporal elasticity of substitution across different varieties of intermediate goods. Solving cost minimization problem for the final goods firms in each sector yields the demand for intermediate goods.

$$C_t(s) = \left(\frac{P_{c,t}(s)}{P_{c,t}} \right)^{-\varepsilon_c} C_t \tag{10}$$

$$I_t(s) = \left(\frac{P_{i,t}(s)}{P_{i,t}} \right)^{-\varepsilon_i} I_t \tag{11}$$

with $P_{j,t}(s)$ the price of intermediate good s in sector j. Aggregate price index in sector j can be characterized by imposing the zero-profit condition:

$$P_{j,t} = \left[\int_0^1 P_{j,t}(s)^{1-\varepsilon_j} ds \right]^{\frac{1}{1-\varepsilon_j}} \tag{12}$$

Intermediate goods firms

In each sector, Intermediate goods producing firms are monopolistically competitive producers of differentiated products. Intermediate goods in each sector is produced according to a Cobb-Douglas production function defined below.

$$C_t(s) = A_t z_{c,t} K_{c,t}(s)^{\alpha} (N_{c,t}(s))^{1-\alpha} - A_t z_{c,t} F_c \tag{13}$$

$$I_t(s) = A_t z_{i,t} K_{i,t}(s)^{\alpha} (N_{i,t}(s))^{1-\alpha} - A_t z_{i,t} F_i \tag{14}$$

where $K_{j,t}(s)$ and $N_{j,t}(s)$ denote capital and labor employed by type s intermediate good producing firm in sector j. F_j denotes the 'fixed cost', which makes it possible for the firms to earn zero profit, and hence makes the capital income share in nonstochastic steady state equal to α. We define aggregate and sectoral productivity shocks separately. A_t is an aggregate total factor productivity (TFP) shock. $z_{j,t}$ is sectoral productivity shock in the sector j. Both shocks follow mean zero AR(1) processes in log.

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{15}$$

$$\log z_{c,t} = \rho_c \log z_{c,t-1} + \epsilon_{c,t} \tag{16}$$

$$\log z_{i,t} = \rho_i \log z_{i,t-1} + \epsilon_{i,t} \tag{17}$$

Intermediate goods firms demand capital and labor given wage and rental rate of capital. Factor demands of intermediate goods firms in each sector are as follows:

$$(1-\tau_c) w_t = \psi_{c,t}(1-\alpha) A_t z_{c,t} K_{c,t}^{\alpha} N_{c,t}^{-\alpha} \tag{18}$$

$$(1-\tau_i) w_t \frac{1}{q_t} = \psi_{i,t}(1-\alpha) A_t z_{i,t} K_{i,t}^{\alpha} N_{i,t}^{-\alpha} \tag{19}$$

$$(1-\tau_c) r_t = \psi_{c,t} \alpha A_t z_{c,t} K_{c,t}^{\alpha-1} N_{c,t}^{1-\alpha} \tag{20}$$

$$(1-\tau_i) r_t \frac{1}{q_t} = \psi_{i,t} \alpha A_t z_{i,t} K_{i,t}^{\alpha-1} N_{i,t}^{1-\alpha} \tag{21}$$

where $q_t = \frac{P_{i,t}}{P_{c,t}}$, the relative price of investment good. We adopt price stickiness a la Calvo (1983). Specifically, each period t and in each sector j, only a fraction θ_j of randomly selected intermediate goods producing firms are not allowed to reset their prices. The remaining $1-\theta_j$ firms choose their prices optimally. An intermediate goods producing firm resetting its price in period t in sector j maximizes the present value of expected future real profit subject to the demand schedule (5) and (6).

$$\max E_t \sum_{\nu=0}^{\infty} (\beta\theta_j)^{\nu} \frac{\lambda_{t+\nu}}{\lambda_t} \frac{P_{j,t}}{P_{j,t+\nu}} \left[P_{j,t}(s)^* Y_{j,t+\nu}(s) - (1-\tau_j) \left(R^k_{t+\nu} K_{t+\nu}(s) - W_{t+\nu} N_{t+\nu}(s) \right) \right]$$

The first order condition for optimal price setting problem can be expressed recursively in a following way:

$$\pi^*_{c,t} = \frac{\varepsilon_c}{\varepsilon_c - 1} \pi_{c,t} \frac{x^2_{c,t}}{x^1_{c,t}} \tag{22}$$

$$x^1_{c,t} = \lambda_t C_t + (\theta_c \beta) E_t \pi^{\varepsilon_c - 1}_{c,t+1} x^1_{c,t+1} \tag{23}$$

$$x^2_{c,t} = \lambda_t \psi_{c,t} C_t + (\theta_c \beta) E_t \pi^{\varepsilon_c}_{c,t+1} x^2_{c,t+1} \tag{24}$$

$$\pi^{1-\varepsilon_c}_{c,t} = (1-\theta_c) {\pi^*_{c,t}}^{1-\varepsilon_c} + \theta_c \tag{25}$$

$$\pi^*_{i,t} = \frac{\varepsilon_i}{\varepsilon_i - 1} \pi_{i,t} \frac{x^2_{i,t}}{x^1_{i,t}} \tag{26}$$

$$x^1_{i,t} = \lambda_t I_t + (\theta_i \beta) E_t \pi^{\varepsilon_i - 1}_{i,t+1} x^1_{i,t+1} \tag{27}$$

$$x^2_{i,t} = \lambda_t \psi_{i,t} I_t + (\theta_i \beta) E_t \pi^{\varepsilon_i}_{i,t+1} x^2_{i,t+1} \tag{28}$$

$$\pi^{1-\varepsilon_i}_{i,t} = (1-\theta_i) {\pi^*_{i,t}}^{1-\varepsilon_i} + \theta_i \tag{29}$$

Where $\pi^*_{j,t}(s)$ is reset price inflation, $\psi_{j,t}$ is real marginal cost in sector j, and λ_t is a Lagrange multiplier attachd to household's budget constraint. In a symmetric equilibrium, all firms face same marginal cost. Hence, $\psi_{j,t}(s) = \psi_{j,t}$. Note that when monopolistic competition is the only distortion, firms set their prices as a fixed markup times the nominal marginal cost. To ignore the distortions arising from monopolistic competition, subsidies are given to intermediate goods producing firms. These subsidies are supported by lump-sum taxation T_t collected from households. Subsidies are given to a firm's marginal cost at the rate of $\frac{1}{\varepsilon_j}$.

2.1.3 Fiscal and Monetary Policy

The fiscal authority runs a balanced budget, which implies

$$T_t = \tau_c \Psi_{c,t} C_t + \tau_i \Psi_{i,t} I_t \tag{30}$$

We assume that the monetary authority seeks a targeting rule under full commmitment. Specifically, monetary authority targets the following composite inflation index.

$$\pi_t = \pi^{\alpha_{\pi_c}}_{c,t} \pi^{1-\alpha_{\pi_c}}_{i,t} \tag{31}$$

α_{π_c} captures relative weights between durable and nondurable goods. α_{π_w} captures relative weights between aggregate price inflation and wage inflation. Monetary authority either focuses on the composite inflation or the output gap. Aggregate output is defined as weighted average of sectoral outputs.

$$Y_t = C^{\Phi_c}_t {I_t}^{(1-\Phi_c)} \tag{32}$$

where Φ_c is steady state share of counsumption goods sector. Hence, $\alpha_{\pi_c} = \Phi_c$ means that the monetary authority is weighing sectoral inflation according to the its steady state share in output. Output gap is defined as difference between aggregate output and its flexible-price counterpart.

$$\widehat{Y}_t = \log Y_t - \log Y_t^f \tag{33}$$

2.1.4 Equilibrium and Aggregation

In equilibrium net bond $B_t = 0$. Labor and capital market equilibrium conditions require

$$N_t = N_{c,t} + N_{i,t} \tag{34}$$

$$K_t = K_{c,t} + K_{i,t} \tag{35}$$

Finally, price dispersions arising from nominal rigidities can be expressed as:

$$\Delta_{c,t} = (1-\theta_c)\pi_{c,t}^{*\ -\varepsilon_c}\pi_{c,t}^{\varepsilon_c} + \theta_c \pi_{c,t}^{\varepsilon_c}\Delta_{c,t-1}$$

$$\Delta_{i,t} = (1-\theta_i)\pi_{i,t}^{*\ -\varepsilon_i}\pi_{i,t}^{\varepsilon_i} + \theta_i \pi_{i,t}^{\varepsilon_i}\Delta_{i,t-1}$$

2.2 Model with imperfect factor mobility

We consider a variant of our baseline model in which input factors are not capable of moving across the sectors. Preferences are replaced with the following equation:

$$U(C_t, N_t) = \log C_t - b_1 \frac{\eta}{1+\eta} N_{c,t}^{\ \frac{\eta+1}{\eta}} - b_2 \frac{\eta}{1+\eta} N_{i,t}^{\ \frac{\eta+1}{\eta}} \tag{36}$$

The aggregate capital accumulation equation (1) is replaced by sectoral capitals accumulation equations:

$$K_{c,t+1} = \varphi_t I_{c,t}\left[1 - \frac{\kappa}{2}\left(\frac{I_{c,t}}{I_{c,t-1}} - 1\right)^2\right] + (1-\delta)K_{c,t} \tag{37}$$

$$K_{i,t+1} = \varphi_t I_{i,t}\left[1 - \frac{\kappa}{2}\left(\frac{I_{i,t}}{I_{i,t-1}} - 1\right)^2\right] + (1-\delta)K_{i,t} \tag{38}$$

where $I_t = I_{c,t} + I_{i,t}$. Intermediate firm's factor demand equations are modified as well:

$$(1-\tau_c)w_{c,t} = \psi_{c,t}(1-\alpha)A_t z_{c,t} K_{c,t}^{\alpha} N_{c,t}^{-\alpha} \tag{39}$$

$$(1-\tau_i)w_{i,t}\frac{1}{q_t} = \psi_{i,t}(1-\alpha)A_t z_{i,t} K_{i,t}^{\alpha} N_{i,t}^{-\alpha} \tag{40}$$

$$(1-\tau_c)r_{c,t} = \psi_{c,t}\alpha A_t z_{c,t} K_{c,t}^{\alpha-1} N_{c,t}^{1-\alpha} \tag{41}$$

$$(1-\tau_i)r_{i,t}\frac{1}{q_t} = \psi_{i,t}\alpha A_t z_{i,t} K_{i,t}^{\alpha-1} N_{i,t}^{1-\alpha} \tag{42}$$

where sectoral wage and rental rate are defined instead of aggregate ones. The rest parts of the model are same as our baseline model. A complete set of all equilibrium conditions for this version of model is laid in the appendix.

2.3 Calibration

We calibrate the model to the U.S. economy with the time unit of one quarter. We assume that th eeconomy is in its deterministic steady state in which the inflation is zero. Table 1 shows our baseline calibration. We set a value of 0.98 for the subjective discount factor β. In both sectors, price rigidities are governed by Calvo parameter θ_j. We followed Justiniano et al. (2010) for most of the parameters. Following them, we set Calvo parameters in both sectors to 0.84, assuming symmetric price rigidities across sectors. Frisch elasticity of labor eta was set to 1/3.79, also following Justiniano et al. (2010). In our benchmark model, we did not consider investment adjustment cost, thus setting κ to zero. The reasonable value for this parameters would be 2.84, as suggested by Christiano et al. (2005). Capital income share $alpha$ is set to 0.17, following the estimated value in Justiniano et al. (2010). It is also assumed that $\varepsilon_c = \varepsilon_i = 6$, as commonly found in the business cycle literature.

We assign a value of one for the steady state aggregate labor N for the convenience. Parameter b in factor mobile model and b_1, b_2 in factor immobile model, representing disutility from labor, were set to match this steady state N. The parameter values for persistence and standard deviation of shocks were either adopted from estimated values or assigned as common values in the business cycle literature. The persistence and standard deviation of two investment shocks are the Bayesian estimates of Katayama and Kim (forthcoming). Since we are focusing on those two investment shocks, we assign common values of parameters for the aggregate TFP shock and consumption sector specific shock. Specifically, we assumed highly persistent aggregate TFP shocks and consumption sector specific shocks.

3 Monetary Policy Tradeoffs

We solve the model by computing a second-order approximation of the policy functions around the non-stochastic steady state, following the approach pioneered by Schmitt-Grohé and Uribe (2004). To understand the tradeoff faced by the monetary authority, we analyze the optimal monetary policy (Ramsey policy) in response to the both IST and MEI shock.

We analyze the effect of such shocks in both version of our model. Factor mobility plays an important role in the propagation of the shocks, therby altering the optimal responses of the central bank. While only IST shock generate tradeoffs in the benchmark model, both IST and MEI shocks produce tradeoffs in the model with imperfect factor mobility.

3.1 Baseline model

Impact of IST shock

In our baseline model with perfect factor mobility, only IST shock induces tradeoffs for the central bank in stabilizing composite inflation or output gap. Figure 1 plots the impulse responses of several variables following a positive investment sector specific (IST) shock. When prices are flexible in both sectors, the economy responses to the IST shock with a immediate increase in investment goods production and a fall in consumption goods production, as the relative price drops sharply at the initial period. Investment goods inflation drops significantly in the initial period, while consumption goods inflation remains the same. Since the increase in investment goods production overwhelms the decreases in consumption goods production, aggregate output spikes up and the central bank responses by raising real interest rate. The economy converges to the steady state level as the effect of shock disappears.

When prices are sticky, however, the dynamics of macroeconomic variables are quite different. Even after the positive shock hits the economy, because of the Calvo friction, only a fraction of firms can drop their prices. Accordingly, a U-shape response in the relative price is observed. Because the relative price is not allowed to drop as it would in the flexible price economy, investment goods are produced less than it would be, creating a negative investment sector output gap. In the meantime, price of consumption goods is remaining cheaper, generating positive consumption sector output gap. These two opposite forces are demonstrated in the figure 1. While only investment sector inflation is affected in the flexible economy, the optimal monetary policy in sticky price environment can neither close the consumption goods inflation nor the investment goods inflation. Optimal monetary policy requires real interest rate to fall, because negative investment sector output gap is a lot larger than the positive consumption sector output gap. The above tradeoffs can be understood intuitively by the following equation. Let us define $\widehat{x_t}$ be a log-linearized variable, the log difference between the variable and its steady state counterpart.

$$\widehat{\psi_{c,t}} - \tilde{q}_t = \widehat{\psi_{i,t}} \tag{43}$$

which can be easily derived from the factor demand equations (18) and (19). Note that sectoral Phillips curve in each sector can be characterized as:

$$\pi_{c,t} = \beta E_t \pi_{c,t+1} + \lambda_c \widehat{\psi_{c,t}} \tag{44}$$

$$\pi_{i,t} = \beta E_t \pi_{i,t+1} + \lambda_i \widehat{\psi_{i,t}} \tag{45}$$

Where $\lambda_j = \frac{(1-\theta_j)(1-\beta\theta_j)}{\theta_j}$. Hence, as long as the relative price $\widehat{q_t}$ is independent of the monetary policy regime, monetary authority cannot stabilize both inflation simultaneously.

Proposition 1. *When prices are sticky and factors can move freely across the sectors, relative price $\widehat{q_t}$ path is invariant of monetary policy regime.*

Proof. Assume that both sectors have the same degree of price stikiness, hence $\lambda_c = \lambda_i = \lambda_p$.

Then, combining two sectoral Philips curves yields:

$$\pi_{c,t} - \pi_{i,t} = \beta E_t \left(\pi_{c,t+1} - \pi_{i,t+1}\right) + \lambda_p \widehat{q_t} \tag{46}$$

From equation (44), we can easily derive the following equation:

$$\Delta \tilde{q}_t = \pi_{i,t} - \pi_{c,t} - \Delta \widehat{q_t}^f \tag{47}$$

where x^f denoting the the flexible counterpart of the variable x. Substituting the latter equation into the former equation, we can express $\tilde{q}_t$ only with $\Delta\widehat{q_t}^f$, which is a function of $\Delta\widehat{z_{c,t}}$ and $\Delta\widehat{z_{i,t}}$. Therefore, regardless of the monetary policy regime, the path of relative price depends solely on technological changes.

□

Impact of MEI shock

Unlike the IST shock, MEi shock does not generate any tradeoff when factors are perfectly mobile across sectors. This is due to that the IST shock only accelerate production in the investment sector, while MEI shock affect both sectors symmetrically. As a positive MEI stimulate the economy by making converting investment goods into capital more efficiently. Therefore, as the households having strong incentive to produce investment goods, investment production jumps up as the shock hits. real wage goes up as the labor demand increases, raising the marginal cost of production in both sectors. As a result, both sectoral inflations moves to the same direction (positive), letting the central bank free from stabilization tradeoffs. Real interest rate path is observed to be raised in the initial period to stabilized the boom, and falls gradually as the effect of the shock disappears. Again, this mechanism can be understood by the equation (43). As the relative price $\widehat{q_t}$ does not move and the two marginal costs move to the same direction, the monetary authority can stabilize both sectoral inflation simultaenously. Therefore, in this case, the so-called *divine coincidence* holds both at the sectoral level and aggregate level. By stabilizing sectoral inflation, the sectoral output gap is stabilized simultaneously. Moreover, because stabilizing the inflation in one sector automatically means stabilizing the inflation in the other sector, aggregate output gap is also closed if the central bank focuses on either inflation. Figure 2 demonstrates this by a perfectly overlapped impulse responses of macroeconomic variables.

3.2 Model with imperfect factor mobility

We now turn our attention to the alternative model, which incorporates the idea of imperfect factor mobility. Unlike the benchmark model, even MEI shock can generate tradeoffs in stabilizing inflation. In this subsection, we analyze the impact of the two shocks and investigate the mechanisms which makes the central bank unable to control both sectoral inflations at the same time.

Impact of IST shock

The effects of IST shock when input factors are not freely mobile across sectors are similar to the one in the benchmark model, since the fundamental mechanism through which shock is propagated remains unchanged. However, the magnitude of the tradeoff becomes smaller relative to the benchmark model, because the relative price distortion is not as severe as in the benchmark model. Figure 3 shows the impulse responses of selected macroeconomic variables. With factors tied up to a specific sector, relative price of investment is now allowed to drop immediately even in the *flexible* price economy due to limited mobility of production factors. When an IST shock arrives to the investment sector, the extent of expansionary effect is limited because labor forces in consumption sector are not allowed to move in to the investment sector. Therefore, investment goods production peaks one period after the arrival of shock, with more accumulated sectoral capital boosting up the production further.

Less distortion in relative prices allows us to expect that monetary policy facing IST shocks would perform relatively well in this alternative economy. We will confirm our prediction in the following section.

Impact of MEI shock

In the previous section, we showed that the MEI shock does not produce any policy tradeoffs in our benchmark model. This story does not remain as it was, when factor immobility is introduced to the economy. Figure 4 shows the impulse responses of macroeconomic variables toward a MEI shock. When the shock hits the economy, labor demand in the investment sector increases immediately, as the efficiency of converting investments to capital is improved in both sectors. However, since labor forces in consumption sector could not move to the investment sector, wage differential occurs with higher investment sector wage and lower consumption sector wage. As a result, positive output gap is observed in the investment sector due to lower markup. Since the magnitude of positive investment output gap is so large, the central bank is forced to raise the real interest rate to stabilize the gap.

As a result, when factors are restricted to move across sectors, monetary authority faces a tradeoff that is opposite to the one in the benchmark economy: it has to positive investment sector inflation and negative consumption sector inflation at the same time. The optimal monetary policy lies somewhere between the two, allowing sectoral inflation fluctuation to some extent. In the next section, we evaluate various monetary policy rules and investigate which rule puts more weights on what component.

4 Optimized Policy Rules

We consider five different monetary policies. Two of them are policies in which monetary authority is only concerned in one specific sector. the CPI targeting rule focuses only in the consumption goods sector inflation, ($\alpha_{\pi_c} = 1$) while Investment inflation targeting ruls focuses only in the investment goods sector inflation ($\alpha_{\pi_c} = 0$) . Two othe rules are 'compromised rules', which targets a weighted average of the two sectoral inflations. the *Harmonic mean* targeting rule gives each sectoral inflation its share of steady state of output, and the *Optimal weight* targeting rule set the optimal weight on each sectoral inflation which maximizes welfare. Finally, aggregate output gap targeting rule was also analyzed.

Measuring welfare cost

We conduct policy evaluation by comparing welfare costs of each particular monetary policy. The time-invariant equilibrium processes of the Ramsey optimal allocation was used as a reference for the policy evaluation. Welfare costs are calculated with the method proposed by Schmitt-Grohé and Uribe (2007). Specifically, we set the welfare level associated with Ramsey allocation as:

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^r, N_t^r) \tag{48}$$

While welfare level under a particular monetary policy can be expressed as:

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^I, N_t^I) \tag{49}$$

We defined Ω as the welfare cost of adopting policy regime I instead of Ramsey policy, measured as a fraction of lifetime consumption. In other words, Ω is the fraction of the consumption process that a household would be willing to give up to be as well off under regime I as under the Ramsey regime.

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^I, N_t^I) = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\Omega)C_t^r, N_t^r) \tag{50}$$

We consider five different monetary policies. *CPI Targeting* is a conventional monetary policy that only focuses on consumption sector inflation. *I-inflation Targeting* rule is the exact opposite of *CPI Targeting*, which only considers investment sector inflation. *Optimal Weight Targeting* rule is a compromise to those two approaches, which gives weights to each sectoral inflation so as to minimize the welfare loss. *Harmonic Mean Targeting* rule is simliar to the former, but instead of determining relative weight according to the welfare criteria, it uses steady state share of each sector, i.e, gives consumption sector inflation Φ_c and investment sector inflation $1-\Phi_c$. Finally, the *Output Gap Targeting* rule targets output gap $\tilde{Y}_t$. All those policies were compared to the optimal (Ramsey) policy path as a reference.

4.1 Performances of targeting rules: Benchmark model

Baseline Calibration

We first test the performances of different monetary policy rules in our baseline model, where factors are perfectly mobile across sectors. Table 2 reports the main findings of that exercise. For each policy rule, theoretical standard deviation of aggregate and sectoral output

gaps, sectoral inflation, and aggregate labor are reported. Welfare cost corresponding to each policy rule is reported in the lowest row. We also report the statistics for the optimal policy in the first column, which provides a refence point for comparison. The top panel reports statistics corresponding to the calibration of Frisch elasticity of labor and the depreciation rate of capital, namely $\eta = 1/3.79$ and $\delta = 0.025$. Relative to the benchmark calibration, alternative parameter values are applied in the following panels. In the following panels, we assumed larger value of Frisch elasticity of labor $\eta = 4$ as well as larger value of depreciation rate, $\delta = 0.4$. Since MEi shock does not produce tradeoffs in the baseline model, we focus on the monetary policy performances toward IST shock.
For the baseline calibration (top panel), the optimal monetary policy is featured with nearly zero variation of consumption sector inflation, allowing about four times more volatility in investment sector inflation. As a consequence, investment sector output gap is a lot more volatile than the consumption sector output gap. the *Optimal Weight* rule, the one that targets composite inflation with carefully given weight, performs the best among the targeting rules. Under this regime the economy mimics the Ramsey equilibrium with near perfection, generating only negligible welfare cost. Result of targeting investment goods inflation is not good, allowing too much variation of consumption goods inflation.

The monetary authority's tradeoffs are evidently observed from the performances of *I-inflation Targeting* rule and *CPI Targeting* rule. In each of these two rules, the cost of stabilizing one of the two sectoral inflation is the maximum fluctuation of the other sectoral inflation. The magnitude of this fluctuation is same for the two regime, as suggested by equation (43). Since the optimal monetary policy places more emphasis on the consumption sector inflation, the *I-inflation Targeting* rules generates more welfare costs than the *CPI Targeting* rule.

Role of labor supply elasticity

Under the baseline calibration, *Output Gap Targeting* rule is the worst, largely due to the large fluctuation in consumption goods inflation. This result can be understood by the following equations:

$$\tilde{Y}_t \cong \alpha \tilde{K}_t + (1-\alpha)\tilde{N}_t \tag{51}$$

$$\widehat{N_{c,t}} + \frac{1}{\eta}\widehat{N_t} \cong \widehat{\psi_{c,t}} \tag{52}$$

The first equation can be derived by using production functions and the definition of aggregate output. The second equation comes from the household's labor supply curve and factor demand for labor, using that capital-labor ratios $\frac{K_{j,t}}{N_{j,t}}$ are the same in both sectors. The first equation implies that aggregate output gap can be decomposed to sum of the aggregate capital gap and the aggregate labor gap. Because capital is under accumulated regardless of policy regime, aggregate capital gap is always negative in the baseline model, which means positive and persistent aggregate labor gap. Positive labor gap is translated to higher marginal cost in the consumption goods sector, which drives up the consumption sector inflation. This effect is well reported in the Figure 5. Since η is small in our baseline calibration, output gap targeting allows large fluctuation of consumption goods inflation (Panel (4,1)), thereby generating large welfare loss. When η gets larger (the second panel of the Table 2), the welfare cost of output gap also gets smaller. Figure 8 shows that the welfare cost of the output gap targeting rule becomes negligible as the value of η increases.

The labor supply elasticity η not only governs the performance of the output gap targeting rule, but also affects the optimal weight in the composite inflation targeting rules. Because higher value of labor supply elasticity invokes more variability in investment goods production, it becomes optimal to give investment sector more weight. Figure 9 shows the decreasing optimal weight attached to the consumption sector inflation along the values of η. Therefore, as the value of η increases, welfare cost of *Harmonic Mean* targeting rule worsens, because it is concentrating too much on the consumption sector inflation.

4.2 Performances of targeting rules: imperfect factor mobility model

We conduct the same exercise with the alternative model, featuring imperfect factor mobility across sectors. Since both IST and MEI shocks trigger inflation tradeoffs, we investigate the performance of five rules toward both shocks. We first look at the case when the economy was hit by an IST shock.

4.2.1 IST shock

Baseline Calibration

Table 3 reports the performances of five rules in the factor-immobile economy, assuming that the economy was hit by an IST shock. Unlike the economy with perfect factor mobility, all five rules work relatively well, producing very small welfare costs. The *Optimal Weight* rule almost perfectly mimics the optimal monetary policy. In the factor immobile economy, the relation between sectoral output gap and inflations can be understood by the following equations:

$$\tilde{C}_t \cong \alpha \tilde{K_{c,t}} + (1-\alpha)\tilde{N_{c,t}} \tag{53}$$

$$\tilde{I}_t \cong \alpha \tilde{K_{i,t}} + (1-\alpha)\tilde{N_{i,t}} \tag{54}$$

$$\left(1+\frac{1}{\eta}\right)\tilde{N_{c,t}} \cong \widehat{\psi_{c,t}} \tag{55}$$

$$\left(1+\frac{1}{\eta}\right)\tilde{N_{i,t}} \cong \widehat{\psi_{c,t}} \tag{56}$$

The first two equations are from sectoral production functions, and the last two equations can be derived household's sectoral labor supply and sectoral production functions. Moreover, the underaccumulation of capital is negligible in this alternative economy, because relative price distortion is not as severe as it was in the benchmark economy. Using this fact, we can simply assume the sectoral capital gap to zero and write:

$$\tilde{C}_t \cong (1-\alpha)\tilde{N}_{c,t} \tag{57}$$

$$\tilde{I}_t \cong (1-\alpha)\tilde{N}_{i,t} \tag{58}$$

This result imdicates that the form of *sectoral divine coincidence* is possible when factor immobility is introduced to the economy. By stabilizing sectoral inflation, the central bank is automatically stabilized *sectoral* output gap as well. Table 3 shows the evidence of this *sectoral divine coincidence*. When the central bank is targeting CPI inflation, the volatility of consumption goods gap is muted as well. On the other hand, when the central bank is targeting investment goods inflation, the volatility of investment goods gap is largely suppressed as well. Relatively better performance of aggregate output gap targeting can be understood by this reason, too. When factors are not mobile across sectors, aggregate output gap can be expressed as

$$\tilde{Y}_t = \Phi_c\tilde{C}_t + (1-\Phi_c)\tilde{I}_t \tag{59}$$

Targeting only one of the two sectoral output gap may not stabilize the aggregate output gap, for it would allow the other sectoral output gap to fluctuate largely. By targeting aggregate output gap, the monetary authority takes a weighted average of two sectoral output gap targeting, attaining better welfare value than two polar cases.

Role of labor supply elasticity

The labor supply elasticity η plays a similar role as in the benchmark economy. Higher elasticity makes sectoral and aggregate labor gap more volatile, which affects marginal cost and therby inflations. The worsened welfare cost of *I inflation Targeting* and *CPI* targeting rule can be understood by this means. Both rules allow fluctuations of sectoral labor gap in the sector it is not focusing on, thereby allowing larger variation of the other sectoral inflation. For example, *CPI Targeting* rule allows investment sector labor gap and inflation vary a lot, generating large welfare loss as a result.

The labor supply elasticity η affects the optimal weight in the composite inflation targeting rules as well. Figure 10 shows the decreasing optimal weight attached to the consumption sector inflation along the values of η. Therefore, as in the benchmark model welfare cost of *Harmonic Mean* targeting rule worsens as the value of η becomes bigger.

4.2.2 MEI shock

Baseline Calibraiton

Now we turn to the impact of MEI shock, which generates another kind of tradeoff to the monetary authority. Table 4 summarizes the results of five rules toward a MEI shock. As shown in the table, only *Optimal Weight* and *Harmonic Mean* targeting rules generates acceptable level of welfare costs. Among the five rules, the *CPI Targeting* rule performs worst, producing severe level of welfare loss. The real interest rate path of *CPI Targeting* regime in

Figure 7 explain why this rule is performing the worst. The *sectoral divine coincidence* result suggests that focusing on CPI inflation translates to stabilizing consumption sector output gap. Therefore, the monetary authority drops does not raise real interest rate as strongly as it would in the other policy regime, allowing to much variability of investment sector output gap. Therefore, high variation of investment sector output gap and thus aggregate output gap produces heavy welfare losses. Although the same logic can be applied to the *I inflation Targeting* regime, the welfare cost generated under that regime is relatively small because investment sector is a primary source of aggregate volatility. Aggregate output gap targeitng compromises between the two polar targeting rules, but still produces high welfare costs when the value of η is small.

Larger value of η improves the performances of output gap targeting rule as in the IST shock case, but it is observed that the welfare cost of *CPI Targeting* rule becomes even worse with $\eta = 4$. It is because the fluctuations of variables in the investment sectors are too large with higher η.

5 Conclusion

Using a calibrated two-sector DSGE model, we analyzed the optimal response of monetary authority towards two kinds of investment shocks, namely IST and MEI shocks.

We find that IST shock and MEI shock require quite different monetary policy, and practicing monetary policy without precise specification of such shocks would cause a severe welfare losses. When factors are mobile across sectors, IST shock introduces a tradeoff in stabilizing aggregate inflation, forcing the monetary authority to choose a relative weight between the consumption sector inflation and the investment sector inflation. In this case, aggregate output gap targeting generate serious welfare losses. Monetary authority can accomplish optimal level of welfare by targeting a composite inflation, treating investment sector more heavily than its share in aggregate output. MEI shock is turned out to produce no tradeoff in the benchmark economy, keeping the *divine coincidence* result unaffected.

When factor immobility is introduced in the economy, it presents another kind of tradeoffs to the central bank. Unlike the economy with perfect factor mobility, MEI shock carries a tradeoff in stabilizing aggregate inflation, and the conventional CPI targeting delivers substantial welfare losses. Output gap targeting works relatively well when factors are immobile across sectors, with higher η improving its performance further.

Our finding is summarized to that none of the conventionally conducted monetary policy can be successful universally according to the welfare criteria. Therefore, when confronted to an investment shock, the central bank should understand the underlying nature of the investment shocks to conduct a suitable monetary policy. However, in most cases a composite inflation targeting rules work well, attaining welfare level close to the Ramsey optimal policy counterpart.

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A Tables and Figures

Table 1. Benchmark calibration

Parameter	Description	Value
β	Discount factor	0.98
η	Frisch elasticity	1/3.79
κ	Investment adjustment cost parameter	0
δ	Depreciation rate	0.025
α	Capital income share	0.17
N	Steady state aggregate labor	1
b	Disutility from labor (benchmark model)	1.0282
b_1	Disutility from consumption sector labor (immobile model)	2.6113
b_2	Disutility from investment sector labor (immobile model)	330.4776
θ_c	Calvo price stickiness parameter in consumption sector	0.84
θ_i	Calvo price stickiness parameter in investment sector	0.84
ε_c	Elasticity of substitution among consumption good varieties	6
ε_i	Elasticity of substitution among investment good variaeties	6
ρ_A	Persistence of aggregate TFP shock	0.9438
ρ_c	Persistence of consumption-sector productivity shock	0.9
ρ_i	Persistence of investment-sector productivity shock	0.8801
ρ_φ	Persistence parameter of MEI shock	0.8801
σ_A	Standard deviation of aggregate TFP shock	0.01
σ_i	Standard deviation of IST shock	0.0189
σ_c	Standard deviation of consumption-sector productivity shock	0.01
σ_v	Standard deviation of MEI shock	0.0189

Table 2. Volatility and Welfare cost of five rules: IST shock to baseline model

Policy Regime		*Ramsey Policy*	*Output Gap Targeting*	*I Inflation Targeting*	*CPI Targeting*	*Optimal Weight*	*Harmonic Mean*
$\eta = 1/3.79$	$\delta = 0.025$						
	$\sigma(\tilde{Y}_t)$	0.0064	0.0000	0.0032	0.0080	0.0066	0.0063
	$\sigma(\tilde{C}_t)$	0.0198	0.0171	0.0186	0.0200	0.0196	0.0195
	$\sigma(\tilde{I}_t)$	0.1792	0.1354	0.1321	0.1961	0.1833	0.1800
	$\sigma(\pi_{c,t})$	0.0006	0.0049	0.0029	0.0000	0.0006	0.0007
	$\sigma(\pi_{i,t})$	0.0023	0.0031	0.0000	0.0029	0.0023	0.0022
	$\sigma(\tilde{N}_t)$	0.0030	0.0037	0.0024	0.0049	0.0036	0.0033
	Welfare Cost (%)		0.1975	0.0559	0.0043	0.0004	0.0006
$\eta = 4$	$\delta = 0.025$						
	$\sigma(\tilde{Y}_t)$	0.0120	0.0000	0.0110	0.0352	0.0168	0.0237
	$\sigma(\tilde{C}_t)$	0.0184	0.0190	0.0212	0.0171	0.0168	0.0165
	$\sigma(\tilde{I}_t)$	0.2211	0.1504	0.0726	0.4135	0.2770	0.3282
	$\sigma(\pi_{c,t})$	0.0010	0.0020	0.0029	0.0000	0.0012	0.0007
	$\sigma(\pi_{I,t})$	0.0021	0.0010	0.0000	0.0029	0.0017	0.0022
	$\sigma(\tilde{N}_t)$	0.0104	0.0045	0.0121	0.0339	0.0169	0.0232
	Welfare Cost (%)		0.0167	0.0558	0.0293	0.0055	0.0091

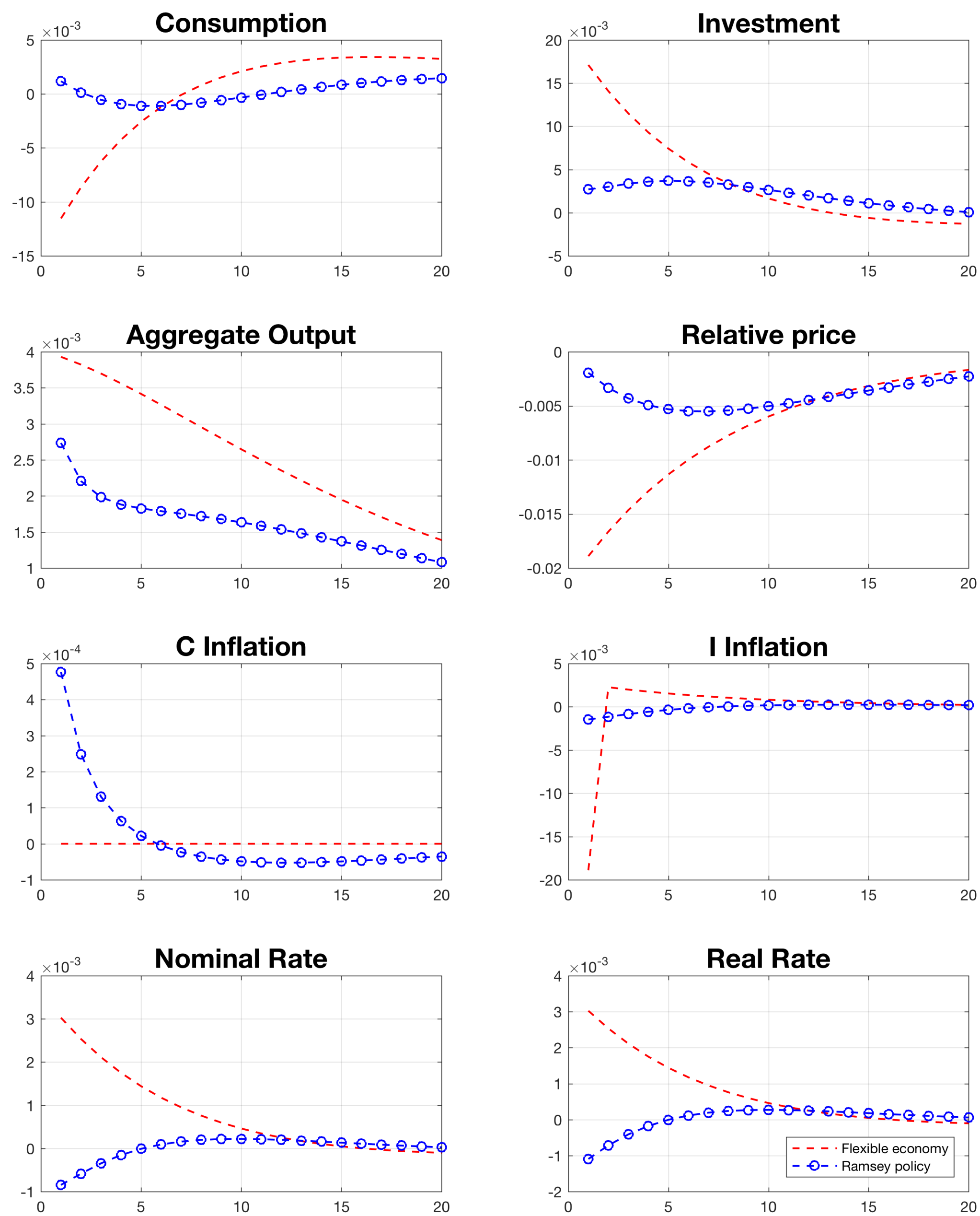
Table 3. Volatility and Welfare cost of five rules: IST shock to immobile model

Policy Regime	*Ramsey Policy*	*Output Gap Targeting*	*I Inflation Targeting*	*CPI Targeting*	*Optimal Weight*	*Harmonic Mean*
$\eta = 1/3.79 \quad \delta = 0.025$						
$\sigma(\tilde{Y}_t)$	0.0008	0.0000	0.0016	0.0016	0.0005	0.0005
$\sigma(\tilde{C}_t)$	0.0014	0.0009	0.0019	0.0004	0.0012	0.0012
$\sigma(\tilde{I}_t)$	0.0063	0.0073	0.0015	0.0138	0.0063	0.0063
$\sigma(\pi_{c,t})$	0.0003	0.0007	0.0011	0.0000	0.0004	0.0004
$\sigma(\pi_{I,t})$	0.0011	0.0013	0.0000	0.0022	0.0011	0.0011
$\sigma(\tilde{N}_{c,t})$	0.0011	0.0008	0.0018	0.0000	0.0010	0.0010
$\sigma(\tilde{N}_{i,t})$	0.0035	0.0057	0.0002	0.0134	0.0041	0.0041
Welfare Cost (%)		0.0036	0.0076	0.0064	0.0001	0.0001
$\eta = 4 \quad \delta = 0.025$						
$\sigma(\tilde{Y}_t)$	0.0029	0.0000	0.0067	0.0145	0.0019	0.0027
$\sigma(\tilde{C}_t)$	0.0051	0.0047	0.0076	0.0032	0.0044	0.0042
$\sigma(\tilde{I}_t)$	0.0381	0.0374	0.0030	0.1300	0.0465	0.0524
$\sigma(\pi_{c,t})$	0.0005	0.0008	0.0016	0.0000	0.0006	0.0005
$\sigma(\pi_{I,t})$	0.0015	0.0011	0.0000	0.0027	0.0014	0.0015
$\sigma(\tilde{N}_{c,t})$	0.0043	0.0044	0.0075	0.0004	0.0038	0.0034
$\sigma(\tilde{N}_{i,t})$	0.0306	0.0318	0.0022	0.1257	0.0384	0.0437
Welfare Cost (%)		0.0027	0.0174	0.0411	0.0011	0.0015

Table 4. Volatility and Welfare cost of five rules: MEI shock to immobile model

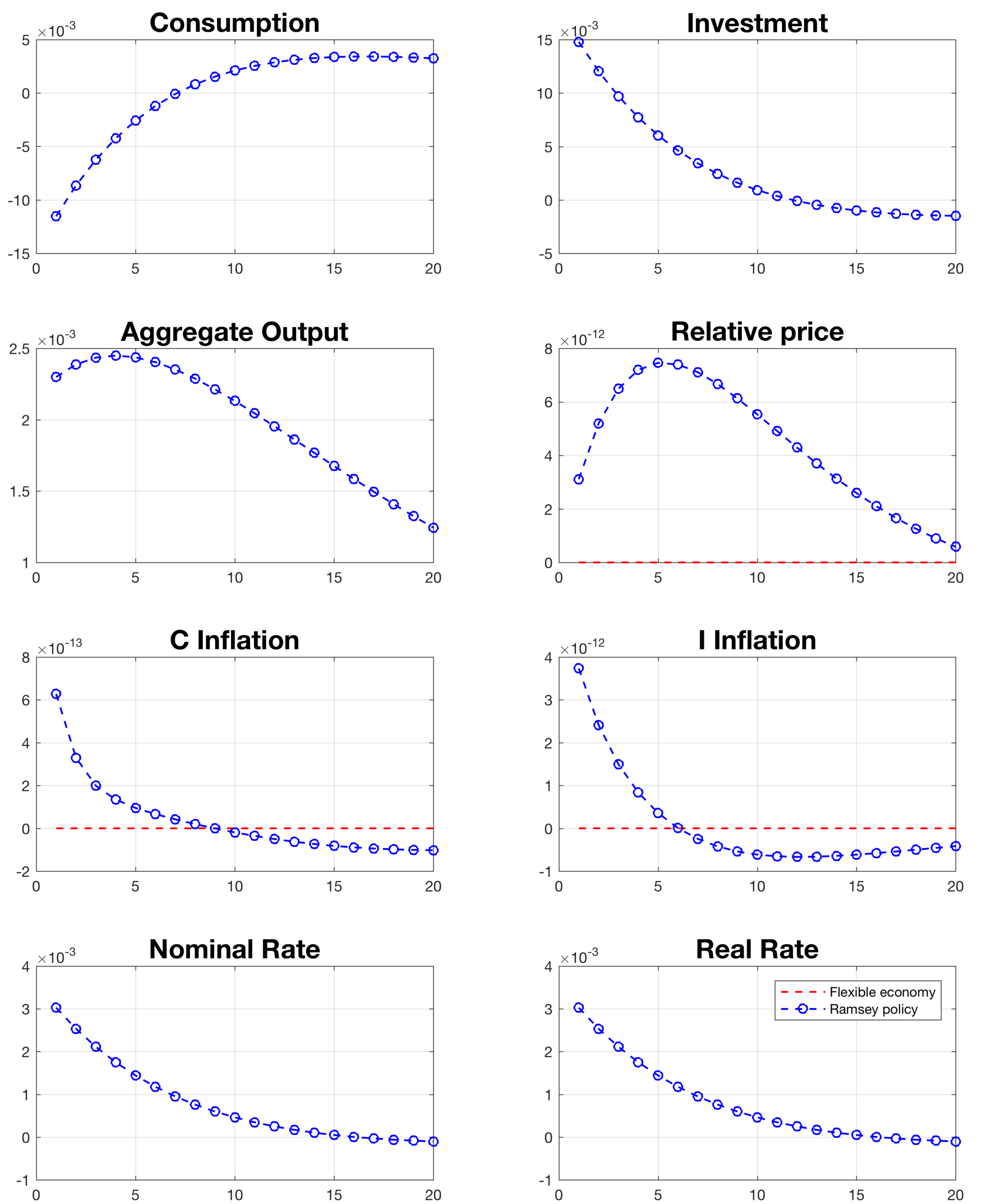
Policy Regime		*Ramsey Policy*	*Output Gap Targeting*	*I Inflation Targeting*	*CPI Targeting*	*Optimal Weight*	*Harmonic Mean*
$\eta = 1/3.79$	$\delta = 0.025$						
	$\sigma(\tilde{Y}_t)$	0.0070	0.0000	0.0115	0.0138	0.0060	0.0041
	$\sigma(\tilde{C}_t)$	0.0108	0.0068	0.0136	0.0026	0.0104	0.0092
	$\sigma(\tilde{I}_t)$	0.0355	0.0535	0.0092	0.1222	0.0317	0.0400
	$\sigma(\pi_{c,t})$	0.0021	0.0030	0.0047	0.0000	0.0025	0.0020
	$\sigma(\pi_{I,t})$	0.0051	0.0081	0.0000	0.0151	0.0046	0.0059
	$\sigma(\tilde{N}_{c,t})$	0.0100	0.0065	0.0130	0.0001	0.0097	0.0086
	$\sigma(\tilde{N}_{i,t})$	0.0218	0.0502	0.0015	0.1225	0.0233	0.0328
	Welfare Cost (%)		0.1024	0.1180	0.5097	0.0057	0.0109
$\eta = 4$	$\delta = 0.025$						
	$\sigma(\tilde{Y}_t)$	0.0088	0.0000	0.0137	0.0625	0.0060	0.0050
	$\sigma(\tilde{C}_t)$	0.0135	0.0117	0.0159	0.0083	0.0132	0.0109
	$\sigma(\tilde{I}_t)$	0.0640	0.0929	0.0087	0.5622	0.0599	0.1205
	$\sigma(\pi_{c,t})$	0.0010	0.0011	0.0018	0.0000	0.0012	0.0008
	$\sigma(\pi_{I,t})$	0.0016	0.0018	0.0000	0.0077	0.0012	0.0023
	$\sigma(\tilde{N}_{c,t})$	0.0126	0.0113	0.0157	0.0011	0.0126	0.0100
	$\sigma(\tilde{N}_{i,t})$	0.0519	0.0874	0.0055	0.5631	0.0513	0.1102
	Welfare Cost (%)		0.0130	0.0167	0.8682	0.0028	0.0191

Figure 1. Ramsey path to IST shock: Economy with perfect factor mobiltiy



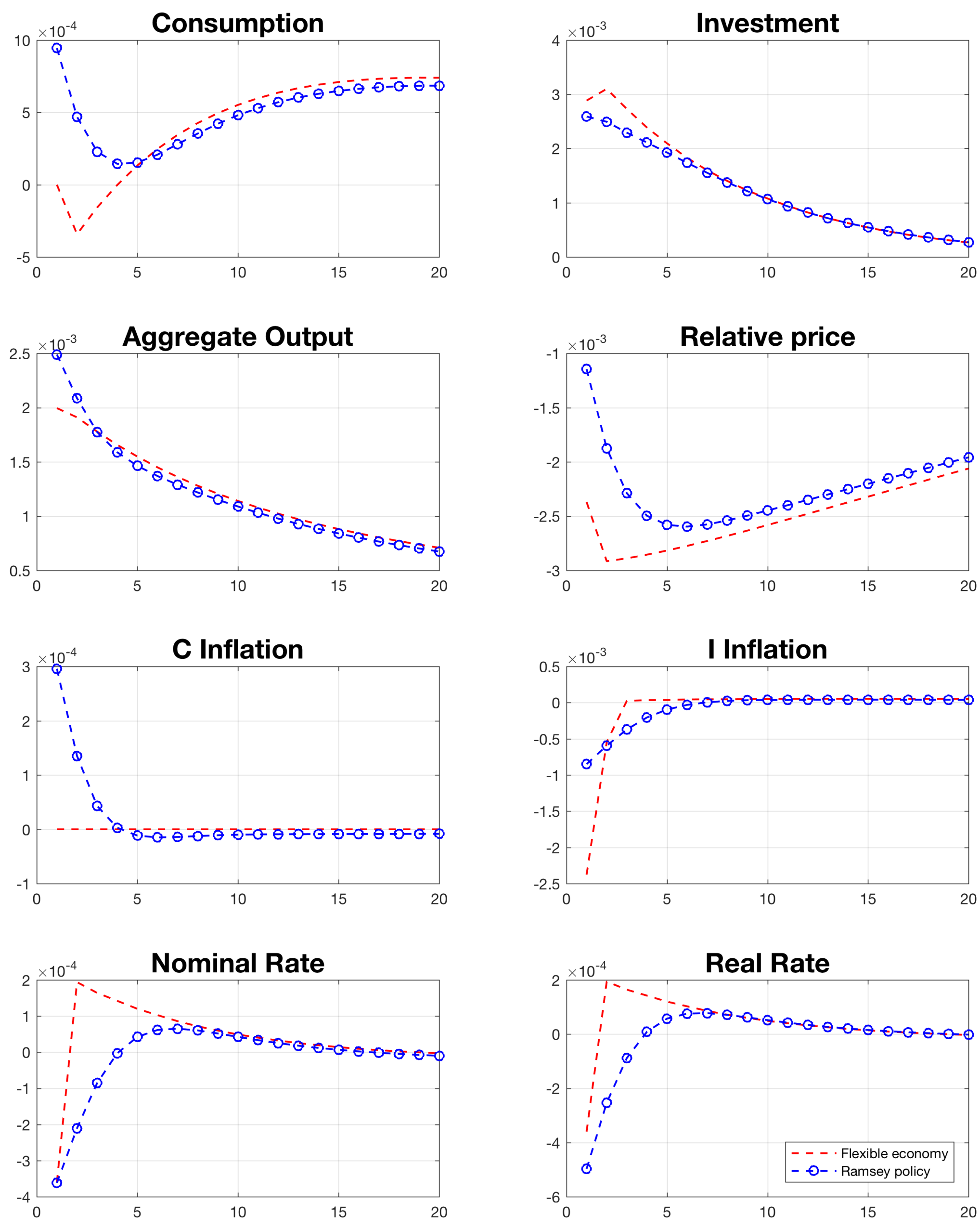
Note: Responses of flexible price economy is denoted by red dotted line. Ramsey optimal policy path is denoted by blue dotted line with circles.

Figure 2. Ramsey path to MEI shock: Economy with perfect factor mobiltiy



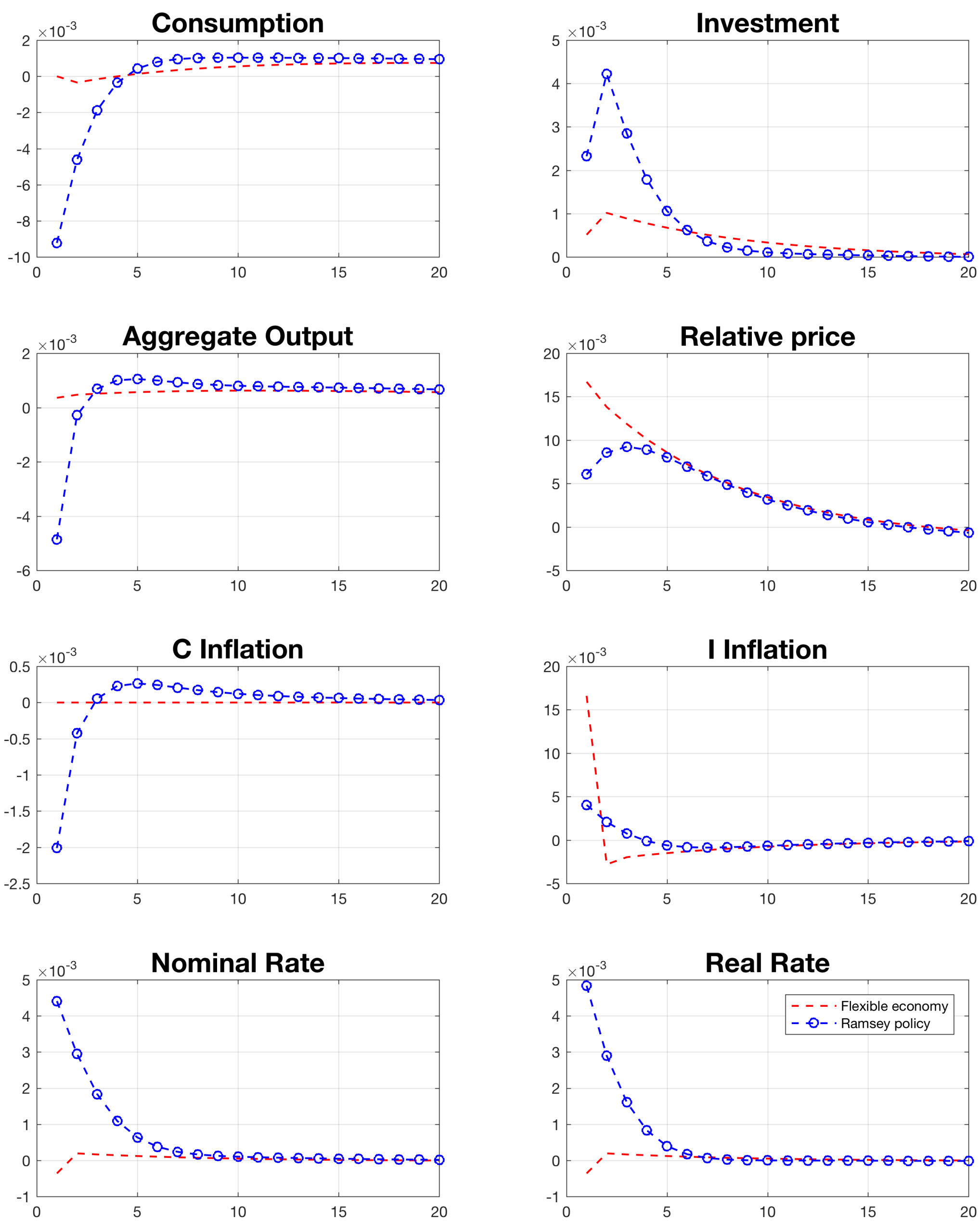
Note: Responses of flexible price economy is denoted by red dotted line. Ramsey optimal policy path is denoted by blue dotted line with circles.

Figure 3. Ramsey path to IST shock: Economy with imperfect factor mobiltiy



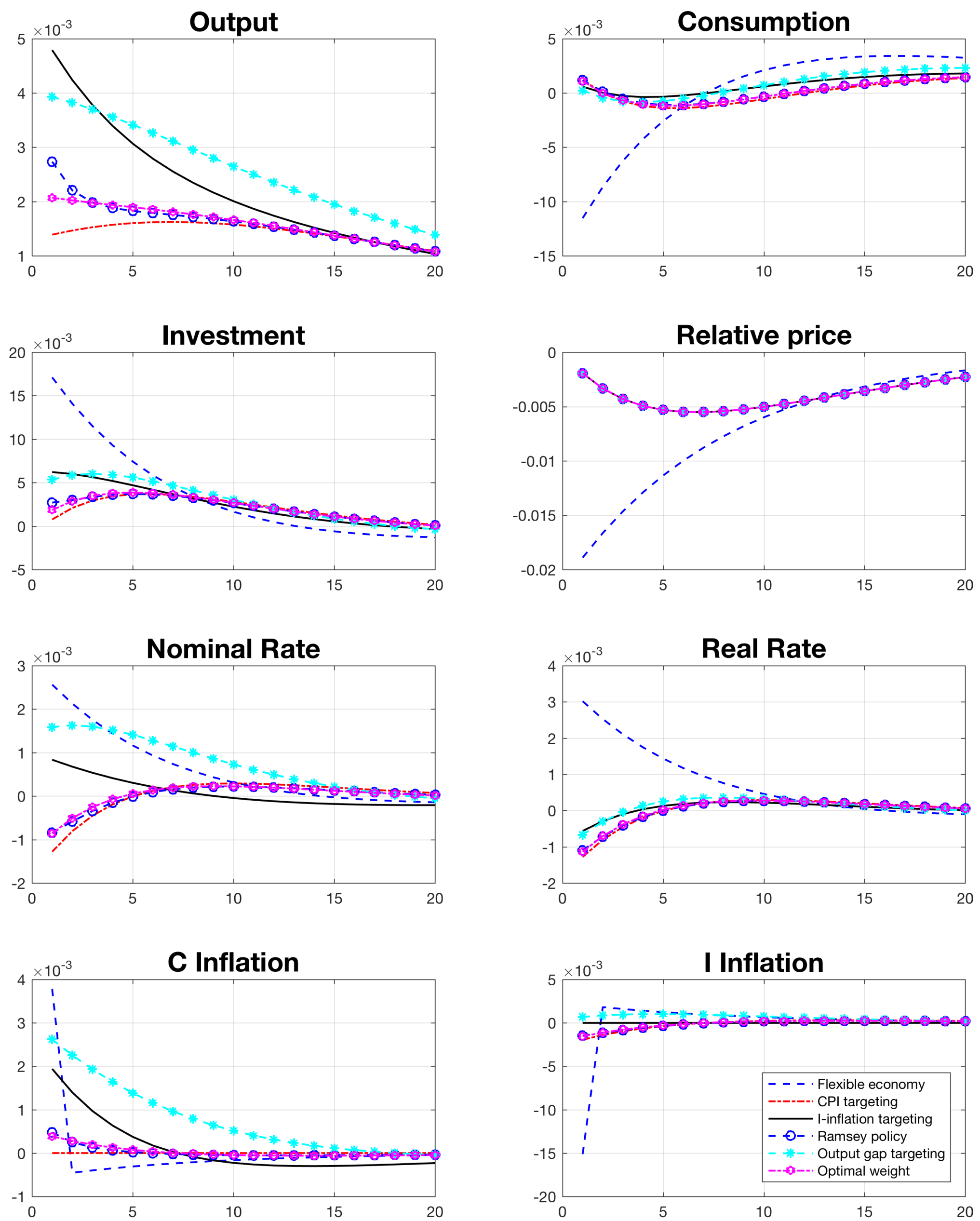
Note: Responses of flexible price economy is denoted by red dotted line. Ramsey optimal policy path is denoted by blue dotted line with circles.

Figure 4. Ramsey path to MEI shock: Economy with imperfect factor mobiltiy



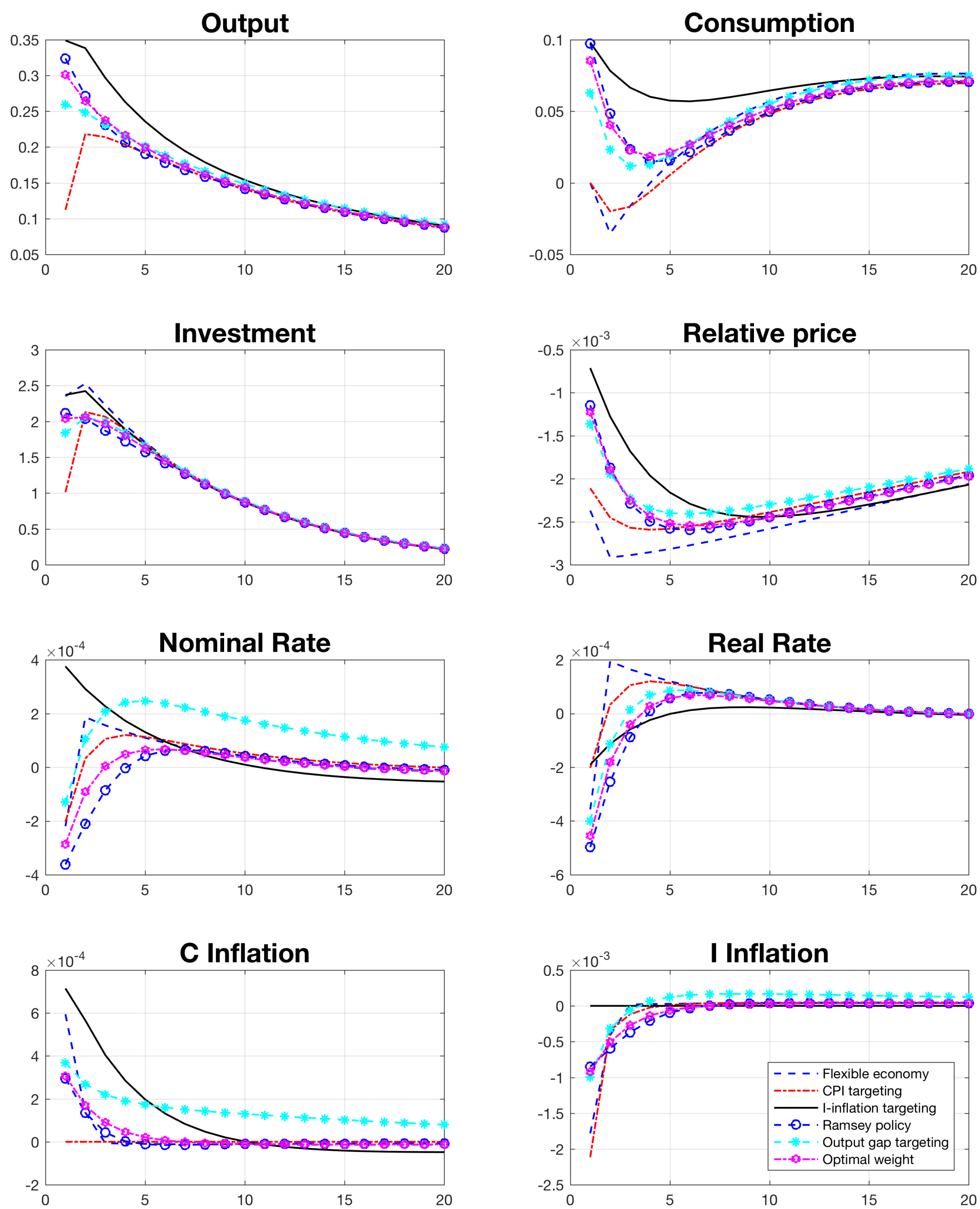
Note: Responses of flexible price economy is denoted by red dotted line. Ramsey optimal policy path is denoted by blue dotted line with circles.

Figure 5. Targeting rules to IST shock: Economy with perfect factor mobiltiy



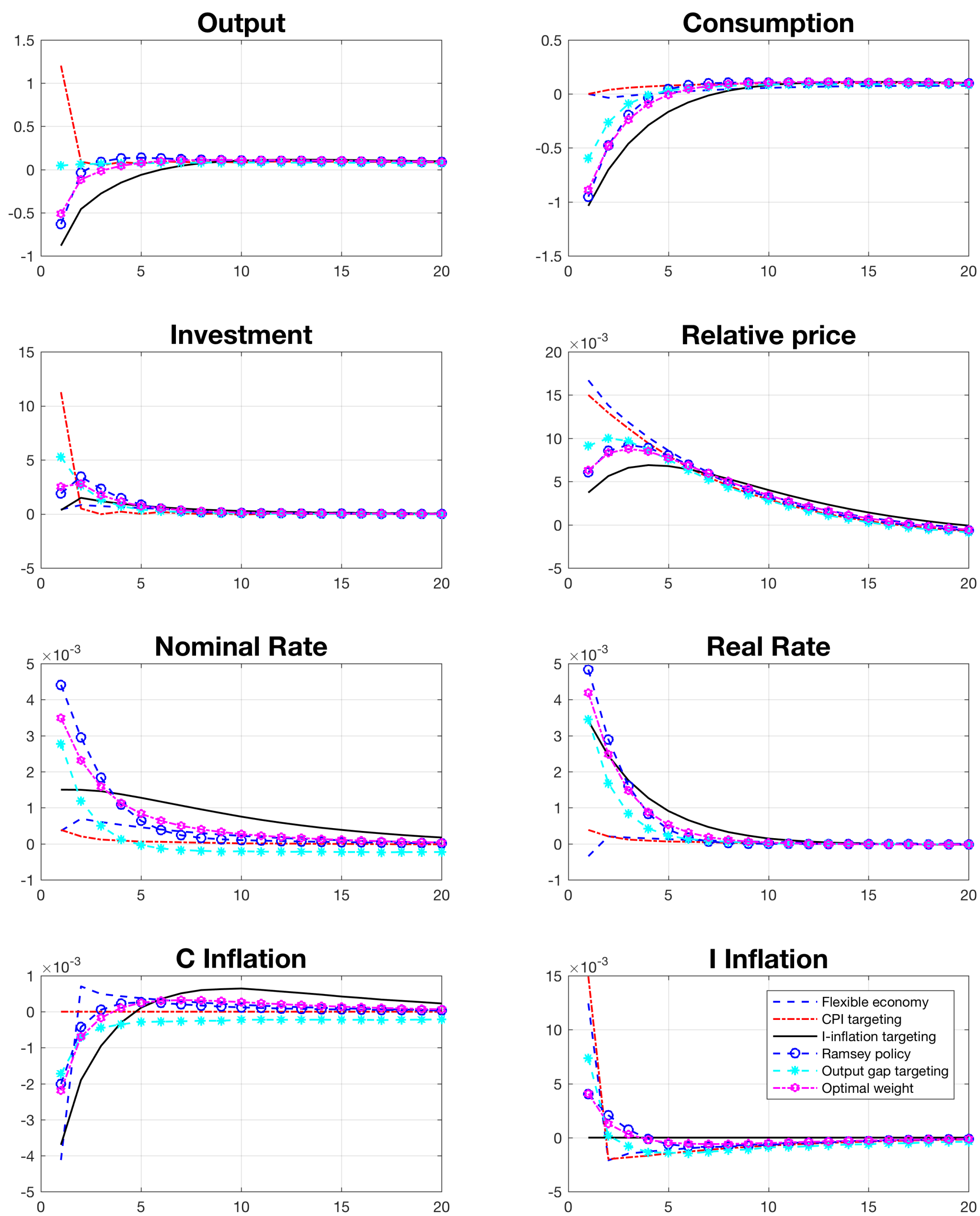
Note: Responses of flexible price economy is denoted by blue dashed line. Ramsey optimal policy path is denoted by blue dotted line with circles. CPI targeting is denoted by red dotted line. Output gap targeting is denoted with cyan dotted line with asteriks, and the Optimal weight policy is denoted by red dashed line with circles.

Figure 6. Targeting rules to IST shock: Economy with imperfect factor mobilty



Note: Responses of flexible price economy is denoted by blue dashed line. Ramsey optimal policy path is denoted by blue dotted line with circles. CPI targeting is denoted by red dotted line. Output gap targeting is denoted with cyan dotted line with asteriks, and the Optimal weight policy is denoted by red dashed line with circles.

Figure 7. Targeting rules to MEI shock: Economy with imperfect factor mobiltiy



Note: Responses of flexible price economy is denoted by blue dashed line. Ramsey optimal policy path is denoted by blue dotted line with circles. CPI targeting is denoted by red dotted line. Output gap targeting is denoted with cyan dotted line with asteriks, and the Optimal weight policy is denoted by red dashed line with circles.

Figure 8. Welfare cost of output gap targeting with varying eta

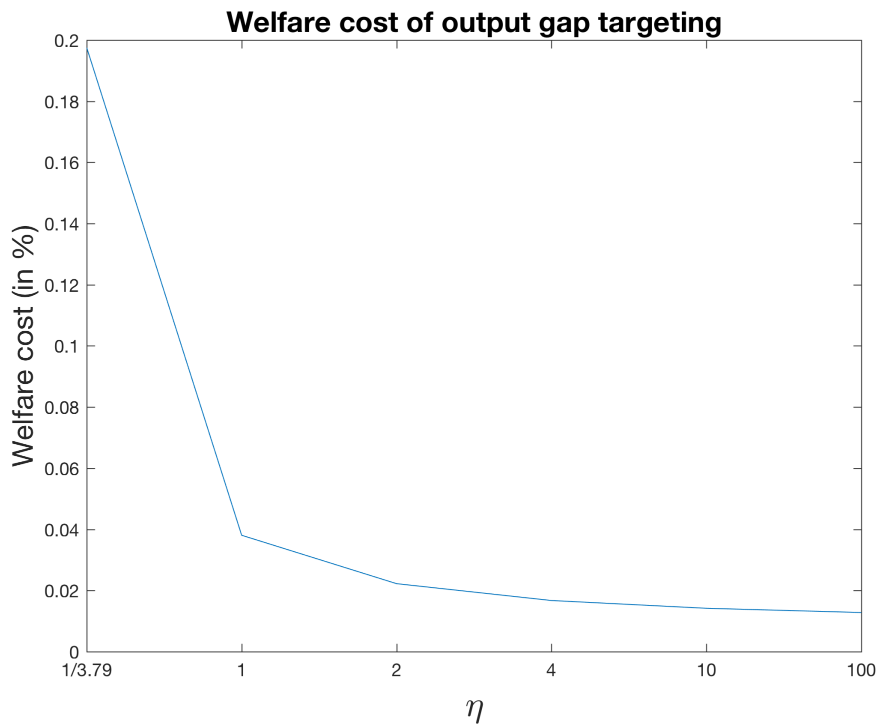


Figure 9. Optimal weight toward IST shock: Economy with perfect factor mobility

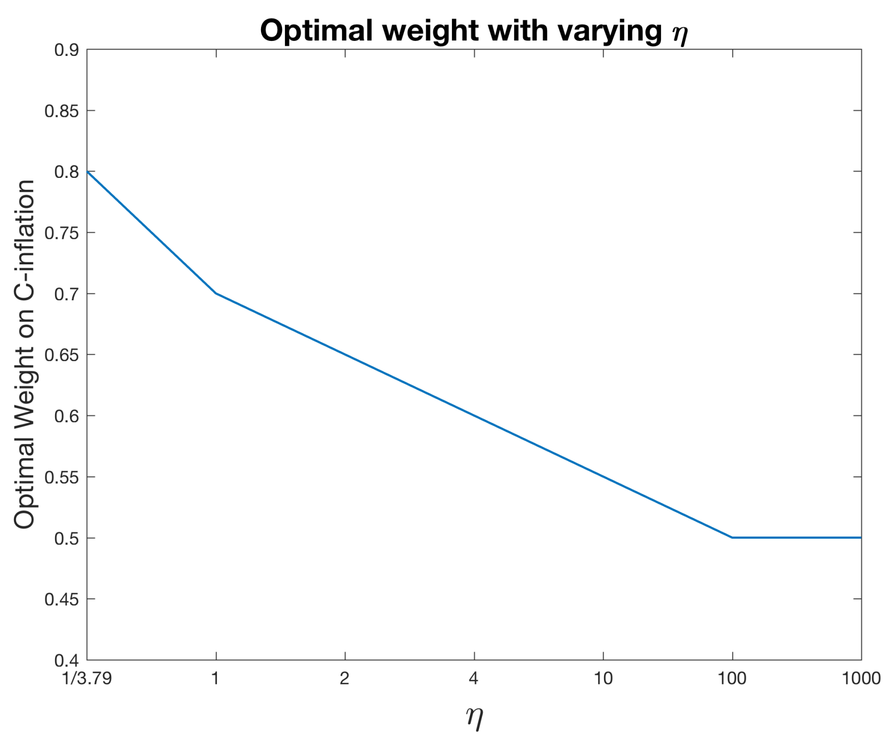


Figure 10. Optimal weight toward IST shock: Economy with imperfect factor mobility

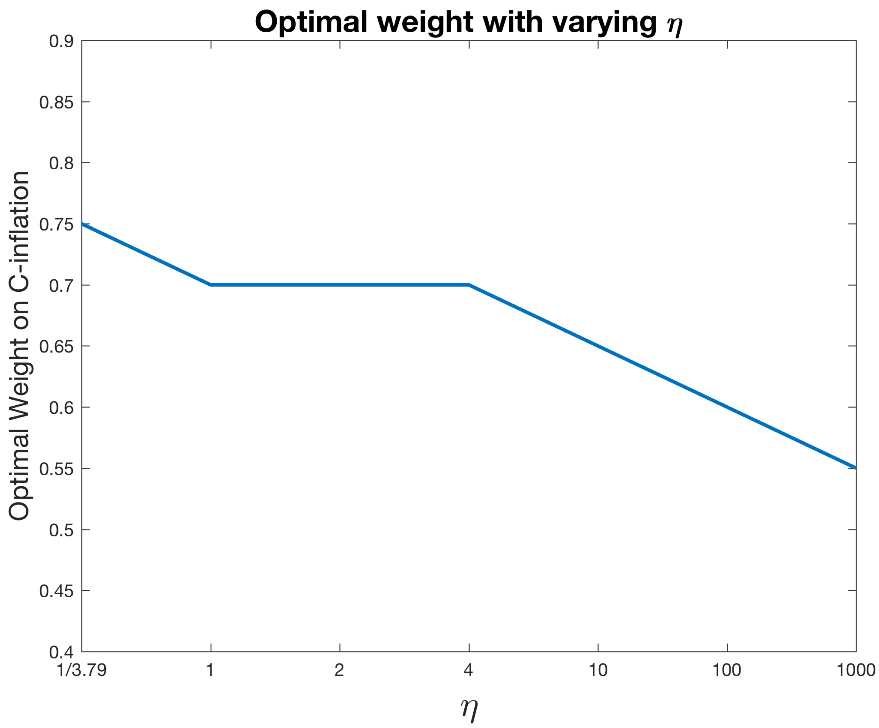
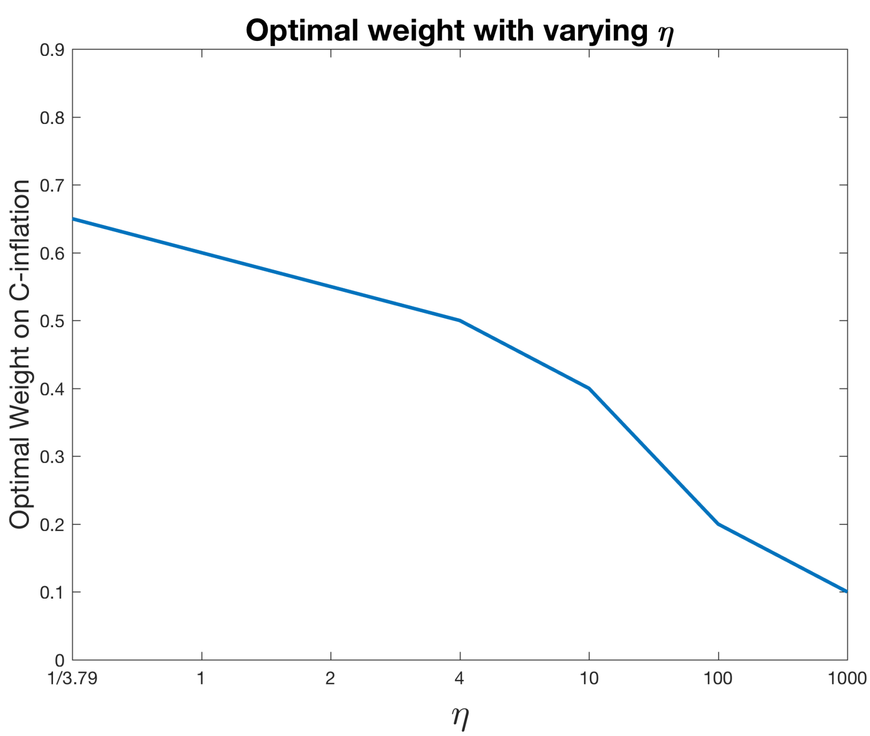


Figure 11. Optimal weight toward MEI shock: Economy with imperfect factor mobility



B Full set of equilibrium conditions for economy with perfect factor mobility

Households

$$\frac{1}{C_t} = \lambda_t \tag{60}$$

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{c,t+1}} \right) \tag{61}$$

$$\lambda_t q_t = \mu_t \varphi_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \kappa \beta E_t \left[\mu_{t+1} \varphi_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \tag{62}$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} r_{t+1} + (1-\delta) \mu_{t+1} \right] \tag{63}$$

$$K_{t+1} = \varphi_t I_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] + (1-\delta) K_t \tag{64}$$

$$MRS_t = b C_t N_t^{\frac{1}{\eta}} \tag{65}$$

$$\frac{1}{C_t} MRS_t = w_t \lambda_t \tag{66}$$

Firms

$$(1-\tau_c) w_t = \psi_{c,t} (1-\alpha) A_t z_{c,t} K_{c,t}^{\alpha} N_{c,t}^{-\alpha} \tag{67}$$

$$(1-\tau_i) w_t \frac{1}{q_t} = \psi_{i,t} (1-\alpha) A_t z_{i,t} K_{i,t}^{\alpha} N_{i,t}^{-\alpha} \tag{68}$$

$$(1-\tau_c) r_t = \psi_{c,t} \alpha A_t z_{c,t} K_{c,t}^{\alpha-1} N_{c,t}^{1-\alpha} \tag{69}$$

$$(1-\tau_i) r_t \frac{1}{q_t} = \psi_{i,t} \alpha A_t z_{i,t} K_{i,t}^{\alpha-1} N_{i,t}^{1-\alpha} \tag{70}$$

Price setting

$$x^1_{c,t} = \lambda_t Y_{c,t} + (\theta_c \beta) E_t \pi^{\varepsilon_c - 1}_{c,t+1} x^1_{c,t+1} \tag{71}$$

$$x^2_{c,t} = \lambda_t \psi_{c,t} Y_{c,t} + (\theta_c \beta) E_t \pi^{\varepsilon_c}_{c,t+1} x^2_{c,t+1} \tag{72}$$

$$\pi^*_{c,t} = \frac{\varepsilon_c}{\varepsilon_c - 1} \pi_{c,t} \frac{x^2_{c,t}}{x^1_{c,t}} \tag{73}$$

$$x^1_{i,t} = \lambda_t Y_{i,t} + (\theta_i \beta) E_t \pi^{\varepsilon_i - 1}_{i,t+1} x^1_{i,t+1} \tag{74}$$

$$x^2_{i,t} = \lambda_t \psi_{i,t} Y_{i,t} + (\theta_i \beta) E_t \pi^{\varepsilon_i}_{i,t+1} x^2_{i,t+1} \tag{75}$$

$$\pi^*_{i,t} = \frac{\varepsilon_i}{\varepsilon_i - 1} \pi_{i,t} \frac{x^2_{i,t}}{x^1_{i,t}} \tag{76}$$

$$\pi^{1-\varepsilon_c}_{c,t} = (1 - \theta_c) {\pi^*_{c,t}}^{1-\varepsilon_c} + \theta_c \tag{77}$$

$$\pi^{1-\varepsilon_i}_{i,t} = (1 - \theta_i) {\pi^*_{i,t}}^{1-\varepsilon_i} + \theta_i \tag{78}$$

Monetary policy

$$tax_t = \tau_c \psi_{c,t} Y_{c,t} + \tau_i \psi_{i,t} Y_{i,t} \tag{79}$$

$$\pi_t = \pi^{\alpha_{\pi_c}}_{c,t} \pi^{1-\alpha_{\pi_c}}_{i,t} \tag{80}$$

Equilibrium and Aggragation

$$Y_{c,t} = C_t \tag{81}$$

$$Y_{i,t} = I_t \tag{82}$$

$$N_{c,t} = \frac{1}{\left(\frac{K_t}{N_t}\right)^\alpha} \left(\Delta_{c,t} \frac{Y_{c,t}}{A_t z_{c,t}} + F_c \right) \tag{83}$$

$$N_{i,t} = \frac{1}{\left(\frac{K_t}{N_t}\right)^\alpha} \left(\Delta_{i,t} \frac{Y_{i,t}}{A_t z_{i,t}} + F_i \right) \tag{84}$$

$$\Delta_{c,t} = (1 - \theta_c) {\pi^*_{c,t}}^{-\varepsilon_c} \pi^{\varepsilon_c}_{c,t} + \theta_c \pi^{\varepsilon_c}_{c,t} \Delta_{c,t-1} \tag{85}$$

$$\Delta_{i,t} = (1 - \theta_i) {\pi^*_{i,t}}^{-\varepsilon_i} \pi^{\varepsilon_i}_{i,t} + \theta_i \pi^{\varepsilon_i}_{i,t} \Delta_{i,t-1} \tag{86}$$

$$N_t = N_{c,t} + N_{i,t} \tag{87}$$

$$K_t = K_{c,t} + K_{i,t} \tag{88}$$

$$Y_t = Y^{\Phi_c}_{c,t} Y^{1-\Phi_c}_{i,t} \tag{89}$$

$$q_t = \frac{\pi_{i,t}}{\pi_{c,t}} q_{t-1} \tag{90}$$

C Full set of equilibrium conditions for model with imperfect factor mobility

Households

$$\frac{1}{C_t} = \lambda_t \tag{91}$$

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{c,t+1}} \right) \tag{92}$$

$$\lambda_t q_t = \mu_t \varphi_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \kappa \beta E_t \left[\mu_{t+1} \varphi_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \tag{93}$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} r_{t+1} + (1-\delta) \mu_{t+1} \right] \tag{94}$$

$$K_{t+1} = \varphi_t I_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] + (1-\delta) K_t \tag{95}$$

$$MRS_t = b C_t N_t^{\frac{1}{\eta}} \tag{96}$$

$$\frac{1}{C_t} MRS_t = w_t \lambda_t \tag{97}$$

Firms

$$(1-\tau_c) w_t = \psi_{c,t} (1-\alpha) A_t z_{c,t} K_{c,t}^{\alpha} N_{c,t}^{-\alpha} \tag{98}$$

$$(1-\tau_i) w_t \frac{1}{q_t} = \psi_{i,t} (1-\alpha) A_t z_{i,t} K_{i,t}^{\alpha} N_{i,t}^{-\alpha} \tag{99}$$

$$(1-\tau_c) r_t = \psi_{c,t} \alpha A_t z_{c,t} K_{c,t}^{\alpha-1} N_{c,t}^{1-\alpha} \tag{100}$$

$$(1-\tau_i) r_t \frac{1}{q_t} = \psi_{i,t} \alpha A_t z_{i,t} K_{i,t}^{\alpha-1} N_{i,t}^{1-\alpha} \tag{101}$$

Price setting

$$x^1_{c,t} = \lambda_t Y_{c,t} + (\theta_c \beta)\, E_t \pi^{\varepsilon_c - 1}_{c,t+1} x^1_{c,t+1} \tag{102}$$
$$x^2_{c,t} = \lambda_t \psi_{c,t} Y_{c,t} + (\theta_c \beta)\, E_t \pi^{\varepsilon_c}_{c,t+1} x^2_{c,t+1} \tag{103}$$
$$\pi^*_{c,t} = \frac{\varepsilon_c}{\varepsilon_c - 1} \pi_{c,t} \frac{x^2_{c,t}}{x^1_{c,t}} \tag{104}$$
$$x^1_{i,t} = \lambda_t Y_{i,t} + (\theta_i \beta)\, E_t \pi^{\varepsilon_i - 1}_{i,t+1} x^1_{i,t+1} \tag{105}$$
$$x^2_{i,t} = \lambda_t \psi_{i,t} Y_{i,t} + (\theta_i \beta)\, E_t \pi^{\varepsilon_i}_{i,t+1} x^2_{i,t+1} \tag{106}$$
$$\pi^*_{i,t} = \frac{\varepsilon_i}{\varepsilon_i - 1} \pi_{i,t} \frac{x^2_{i,t}}{x^1_{i,t}} \tag{107}$$
$$\pi^{1-\varepsilon_c}_{c,t} = (1 - \theta_c) {\pi^*_{c,t}}^{1-\varepsilon_c} + \theta_c \tag{108}$$
$$\pi^{1-\varepsilon_i}_{i,t} = (1 - \theta_i) {\pi^*_{i,t}}^{1-\varepsilon_i} + \theta_i \tag{109}$$

Monetary policy

$$tax_t = \tau_c \psi_{c,t} Y_{c,t} + \tau_i \psi_{i,t} Y_{i,t} \tag{110}$$
$$\pi_t = \pi^{\alpha_{\pi_c}}_{c,t} \pi^{1-\alpha_{\pi_c}}_{i,t} \tag{111}$$

Equilibrium and Aggragation

$$Y_{c,t} = C_t \tag{112}$$
$$Y_{i,t} = I_t \tag{113}$$
$$N_{c,t} = \frac{1}{\left(\frac{K_t}{N_t}\right)^\alpha} \left(\Delta_{c,t} \frac{Y_{c,t}}{A_t z_{c,t}} + F_c \right) \tag{114}$$
$$N_{i,t} = \frac{1}{\left(\frac{K_t}{N_t}\right)^\alpha} \left(\Delta_{i,t} \frac{Y_{i,t}}{A_t z_{i,t}} + F_i \right) \tag{115}$$
$$\Delta_{c,t} = (1 - \theta_c) {\pi^*_{c,t}}^{-\varepsilon_c} \pi^{\varepsilon_c}_{c,t} + \theta_c \pi^{\varepsilon_c}_{c,t} \Delta_{c,t-1} \tag{116}$$
$$\Delta_{i,t} = (1 - \theta_i) {\pi^*_{i,t}}^{-\varepsilon_i} \pi^{\varepsilon_i}_{i,t} + \theta_i \pi^{\varepsilon_i}_{i,t} \Delta_{i,t-1} \tag{117}$$
$$N_t = N_{c,t} + N_{i,t} \tag{118}$$
$$K_t = K_{c,t} + K_{i,t} \tag{119}$$
$$Y_t = Y^{\Phi_c}_{c,t} Y^{1-\Phi_c}_{i,t} \tag{120}$$
$$q_t = \frac{\pi_{i,t}}{\pi_{c,t}} q_{t-1} \tag{121}$$